Give **brief** answers. **SCORE: _____ / 10 PTS**

[a] If $\int_0^4 \frac{1}{x^p}\,dx$ diverges, what can we conclude about p ?

$p \geq 1$

[b] If $0 < r(x) < q(x)$ on $[a, \infty)$, and $\int_a^\infty r(x)\,dx$ converges, what can we conclude about $\int_a^\infty q(x)\,dx$?

NOTHING

[c] If $0 < r(x) < q(x)$ on $[a, \infty)$, and $\int_a^\infty q(x)\,dx$ diverges, what can we conclude about $\int_a^\infty r(x)\,dx$?

NOTHING

Find $\int_0^{\frac{\pi}{4}} \tan^4 z \sec^6 z\,dz$. **SCORE: _____ / 25 PTS**

$u = \tan z$

$du = \sec^2 z\,dz$

$\int_0^{\frac{\pi}{4}} \tan^4 z \sec^4 z \cdot \sec^2 z\,dz$

SEE 7.2 #25

$= \int_0^1 u^4(u^2+1)^2\,du$

$= \int_0^1 (u^8 + 2u^6 + u^4)\,du$

$= \left(\frac{1}{9}u^9 + \frac{2}{7}u^7 + \frac{1}{5}u^5\right)\Big|_0^1$

$= \frac{1}{9} + \frac{2}{7} + \frac{1}{5}$

Determine if $\int_2^\infty \frac{x+1}{\sqrt{x^4-1}}\,dx$ converges or diverges. **SCORE: _____ / 15 PTS**

$\frac{x+1}{\sqrt{x^4-1}} > \frac{x}{\sqrt{x^4-1}} > \frac{x}{\sqrt{x^4}} = \frac{x}{x^2} = \frac{1}{x} > 0$

$\int_2^\infty \frac{1}{x}\,dx$ DIVERGES, SO $\int_2^\infty \frac{x+1}{\sqrt{x^4-1}}\,dx$ DIVERGES

$(p=1)$

SEE MIDTERM 3 REVIEW #2f

AND 7.8 #51

Find $\int_0^1 \frac{1}{x(\ln x)^2}\,dx$. $= \int_0^{\frac{1}{e}} \frac{1}{x(\ln x)^2}\,dx + \int_{\frac{1}{e}}^{1} \frac{1}{x(\ln x)^2}\,dx$

SCORE: _____ / 25 PTS

$$\int_{\frac{1}{e}}^{1} \frac{1}{x(\ln x)^2}\,dx = \lim_{N\to 1^-} \int_{\frac{1}{e}}^{N} \frac{1}{x(\ln x)^2}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \frac{1}{u^2}\,du = -\frac{1}{u}$

$$= \lim_{N\to 1^-} -\frac{1}{\ln x}\Big|_{\frac{1}{e}}^{N}$$

$$= \lim_{N\to 1^-}\left(-\frac{1}{\ln N} - 1\right) \text{ DNE: AS } N \to 1^-$$

$\ln N \to 0^-$

$\frac{1}{\ln N} \to -\infty$

SO $\int_0^1 \frac{1}{x(\ln x)^2}\,dx$ DIVERGES

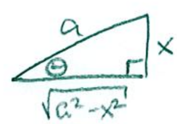
SEE MIDTERM 3 REVIEW #1h

Find $\int x^2\sqrt{a^2 - x^2}\,dx$.

SCORE: _____ / 25 PTS

$x = a\sin\theta \longrightarrow \sin\theta = \frac{x}{a}$

$dx = a\cos\theta\,d\theta$



SEE 7.3 #15

$$\int a^2\sin^2\theta \cdot a\cos\theta \cdot a\cos\theta\,d\theta$$

$$= a^4\int \sin^2\theta\cos^2\theta\,d\theta$$

$$= a^4\int \sin^2\theta(1-\sin^2\theta)\,d\theta$$

$$= a^4\left[-\int \sin^4\theta\,d\theta + \int \sin^2\theta\,d\theta\right]$$

$$= a^4\left[\frac{1}{4}\sin^3\theta\cos\theta - \frac{3}{4}\int \sin^2\theta\,d\theta + \int \sin^2\theta\,d\theta\right]$$

$$= \frac{1}{4}a^4\left[\sin^3\theta\cos\theta + \int \sin^2\theta\,d\theta\right]$$

$$= \frac{1}{4}a^4\left[\sin^3\theta\cos\theta - \frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\theta\right] + C$$

$$= \frac{1}{4}a^4\left[\frac{x^3}{a^3}\frac{\sqrt{a^2-x^2}}{a} - \frac{1}{2}\frac{x}{a}\frac{\sqrt{a^2-x^2}}{a} + \frac{1}{2}\sin^{-1}\frac{x}{a}\right] + C$$

$$= \frac{1}{4}x^3\sqrt{a^2-x^2} - \frac{1}{8}a^2x\sqrt{a^2-x^2} + \frac{1}{8}a^4\sin^{-1}\frac{x}{a} + C$$

Find $\int \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2}\, dx$.

SCORE: _____ / 25 PTS

$$= \int \left(1 - \frac{4}{x^2(x-2)}\right) dx$$

$$\frac{4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

$$4 = Ax(x-2) + B(x-2) + Cx^2$$

$x=0$ $\quad 4 = -2B \rightarrow B = -2$

$x=2$ $\quad 4 = 4C \rightarrow C = 1$

COEF OF x^2 $\quad 0 = A + C \rightarrow A = -1$

SANITY CHECK: $x = -2$

$$\frac{4}{x^3-2x^2} = \frac{4}{-16} = -\frac{1}{4}$$

$$\frac{-1}{-2} + \frac{-2}{4} + \frac{1}{-4} = \frac{1}{2} - \frac{1}{2} - \frac{1}{4} = -\frac{1}{4} \checkmark$$

$$= \int \left(1 + \frac{1}{x} + \frac{2}{x^2} - \frac{1}{x-2}\right) dx$$

$$= x + \ln|x| - \frac{2}{x} - \ln|x-2| + C$$

SEE 7.4 #15

Find $\int \tan^{-1}\sqrt{x}\, dx$.

SCORE: _____ / 25 PTS

$u = \sqrt{x}$

$x = u^2$

$dx = 2u\,du$

SEE 7.5 #21

$$\int 2u \tan^{-1} u \, du = u^2 \tan^{-1} u - \int \frac{u^2}{1+u^2} du$$

U	dV
$\tan^{-1} u$	$2u$
$\frac{1}{1+u^2}$	u^2

$$= u^2 \tan^{-1} u - \int \left(1 - \frac{1}{1+u^2}\right) du$$

$$= u^2 \tan^{-1} u - u + \tan^{-1} u + C$$

$$= x \tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$$